

Transient Crossing of Phantom divide line $w_\Lambda = -1$ under Gauss-Bonnet interaction

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Abstract

Smooth double crossing of the phantom barrier $w_\Lambda = -1$ has been found possible in cosmological model with Gauss-Bonnet-scalar interaction, in the presence of background cold dark matter. Such crossing has been observed to be a sufficiently late time phenomena and independent of the sign of Gauss-Bonnet-scalar interaction. The luminosity distance versus redshift curve shows a perfect fit with the Λ CDM model up to $z = 3.5$.

1 Introduction

The puzzle associated with recent cosmic acceleration, triggered by 70% of dark energy or more [1] is far from being resolved uniquely. In the mean time, cosmologists are being confronted with yet another more intriguing challenge to explain the crossing of the so called phantom divide line ($w_\Lambda = -1$), at sufficiently late time of cosmological evolution. Some recent analysis [1],[2] of the presently available observational data are in favour of the value $w_{de} < -1$, at present., w_{de} being the dark energy equation of state. There are also a lot of evidence all around [3], of a dynamical dark energy equation of state, which has crossed the so called phantom divide line $w_\Lambda = -1$ recently, at the value of red-shift parameter $z \approx 0.2$. Apparently though the problem turns out to be more serious and complicated, but then, the puzzle of crossing the phantom divide line has also rendered some sort of selection rule. Λ CDM-model, which is known to suffer from the disease of fine tuning (see [4] for a comprehensive review) can now be ruled out due to the requirement of a dynamic state parameter. Further, if the analysis of Vikman [5] is correct, then it is not possible to cross the phantom divide line in a single minimally coupled scalar field theory, without violating the stability both at the classical [6] and also at the quantum mechanical levels [7], (though it has recently been inferred [8] that quantum Effects which induce the $w < -1$ phase, are stable in the ϕ^4 model). Thus single minimally coupled scalar field models like quintessence ($w > -1$) and phantom ($w < -1$) are to be kept aside. Consequently, we are now left with some what more complicated models. One of these is a hybrid model, composed of two scalar fields, viz, quintessence and phantom - usually dubbed as quintom model [9]. Other models like non-minimal scalar tensor theory of gravity [10], hessence [11] and models including higher order curvature invariant terms [12] also exist in the literature.

Gauss-Bonnet term is yet another candidate which may be pursued for the purpose. The possibility of crossing the phantom divide line through Gauss-Bonnet interaction has been explored in some recent works [13],[14]. But then, these models are even complicated in the sense that either brane-world scenario [13] or scalar field and matter coupling [14] are invoked. In this article the possibility of smooth crossing of the phantom divide line $w_\Lambda = -1$ has been expatiated simply by introducing Gauss-Bonnet-Scalar coupling term in the 4-dimensional Einstein-Hilbert action.

Gauss-Bonnet term arises naturally as the leading order of the α' expansion of heterotic superstring theory, where, α' is the inverse string tension [15]. Gauss-Bonnet term is topologically invariant and thus does not contribute to the field equations in four dimensions. However, the low energy limit of the string theory gives rise to the dilatonic scalar field which is found to be coupled with various curvature invariant terms [16]. The leading quadratic correction gives rise to Gauss-Bonnet term with a dilatonic coupling [17]. Therefore it is reasonable to consider Gauss-Bonnet interaction in four dimension with dilatonic-scalar coupling. Several works with Gauss-Bonnet-dilatonic coupling are already present in the literature [18]. In particular, important issues like - late time dominance of dark energy after a scaling matter era and thus alleviating the coincidence problem, crossing the

phantom divide line and compatibility with the observed spectrum of cosmic background radiation have also been addressed recently [19].

In a recent work with Gauss-Bonnet interaction [20], a solution in the form $a = a_0 e^{A\sqrt{t}}$ (a being the scale factor, and $A > 0$) has been found to satisfy the field equations with different forms (sum of exponentials, sum of inverse exponentials, sum of powers and even quadratic) of potentials. Solution in a more general form ($a = a_0 e^{At^f}$), $A > 0, 0 < f < 1$, for Einstein's gravity with a minimally coupled scalar field was found in the nineties [21] and was dubbed as intermediate inflation. We [20], on the other hand, observed that such solution depicts a transition from decelerated to accelerated expansion at sufficiently later epoch of cosmic evolution, which asymptotically goes over to de-Sitter expansion. Thus, it appeared that such solution may construct viable cosmological models of present interests. Under this consequence, a comprehensive analysis has been carried out [22] with such solution in the context of a generalized k-essence model. It has been observed that it admits scaling solution with a natural exit from it at a later epoch of cosmic evolution, leading to late time acceleration with asymptotic de-Sitter expansion. The corresponding scalar field has also been found to behave as a tracker field [23], thus avoiding cosmic coincidence problem.

In the present work, we show that Gauss-Bonnet-Dilaton scalar coupling with Einstein's gravity in four dimensions, admits solution in a general form ($a = a_0 e^{At^f}$), $A > 0, 0 < f < 1$, which is viable of crossing the phantom divide line twice, once from above and the other from below in the recent epoch. Since the crossing is transient, so we may conclude that it does not show any pathological behaviour like Big-Rip [6], at least in the classical level.

2 The Model with Gauss-Bonnet Interaction

We start with the following action containing Gauss-Bonnet interaction

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + \frac{\Lambda(\phi)}{8} G(R) - \frac{1}{2} \phi_{,\mu} \phi^{,\mu} - V(\phi) + L_m \right], \quad (1)$$

where,

$$G(R) = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

is the Gauss-Bonnet term which appears in the action with a coupling parameter $\Lambda(\phi)$ and L_m is the matter Lagrangian. For the spatially flat Robertson-Walker space-time

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2],$$

the field equations in terms of the Hubble parameter $H = \frac{\dot{a}}{a}$, are

$$2\dot{H} + 3H^2 = -\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) + 2\Lambda'\dot{\phi}(H\dot{H} + H^3) + (\Lambda'\ddot{\phi} + \Lambda''\dot{\phi}^2)H^2 + p_m\right] = -(p_{de} + p_m), \quad (2)$$

$$3H^2 = \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\Lambda'\dot{\phi}H^3 + \rho_m\right] = (\rho_{de} + \rho_m), \quad (3)$$

in the units $\kappa^2 (= 8\pi G) = \hbar = c = 1$. In our analysis the Gauss-Bonnet scalar interaction plays the role of dark energy, for which suffix (de) has been introduced. Thus, p_{de} and ρ_{de} are the effective pressure and the energy density generated by the Gauss-Bonnet-scalar interaction, while p_m and ρ_m are the pressure and the energy density corresponding to background matter distribution respectively. The background matter satisfies the equation of state,

$$\rho_m = \rho_i a^{-3(1+w_m)}, \quad (4)$$

where, ρ_i is a constant and w_m is the state parameter of the background matter. In addition we have got the ϕ variation equation

$$(\ddot{\phi} + 3H\dot{\phi} + V') = 3\Lambda'H^2(\dot{H} + H^2),$$

which is not an independent equation and will not be required in our analysis. In the above, over-dot and dash (\prime) stand for differentiations with respect to the proper time t and ϕ respectively. Now, in view of equations (2) through (4), we are required to solve for $a, \phi, V(\phi), \Lambda(\phi), p_m$ and ρ_m , which requires three additional assumptions. Firstly, we consider that the Universe is filled with cold dark background matter with equation of state, $p_m = 0$ while the second assumption is the one made previously in [20], viz.,

$$\Lambda' \dot{\phi} = \lambda, \quad (5)$$

where, λ is a constant. This, as indicated in [20] is physically reasonable, since it implies that the Gauss-Bonnet coupling parameter $\Lambda(\phi(t)) = \lambda t$, grows in time to contribute at the later epoch of cosmological evolution. In view of the above assumption the field equations (2) through (4) are expressed as,

$$2\dot{H} + 3H^2 = -[\frac{1}{2}\dot{\phi}^2 - V(\phi) + 2\lambda H\dot{H} + 2\lambda H^3] = -p_{de}, \quad (6)$$

$$3H^2 = [\frac{1}{2}\dot{\phi}^2 + V(\phi) - \lambda H^3 + \rho_m] = (\rho_{de} + \rho_m), \quad (7)$$

and,

$$\rho_m = \rho_i a^{-3}. \quad (8)$$

Now, for our third assumption, we start from the ansatz,

$$H = \frac{f}{nt^{1-f}}, \quad (9)$$

with $0 < f < 1, n = A^{-1} > 0$, which leads to the form of the solution of the scale factor mentioned in the introduction. Thus the complete set of solutions are given by,

$$\begin{aligned} a &= a_0 \exp\left(\frac{t^f}{n}\right); \quad \rho_m = \frac{\rho_i}{a_0^3} \exp\left(-\frac{3}{n}t^f\right); \quad p_{de} = \frac{2f(1-f)}{nt^{2-f}} - \frac{3f^2}{n^2t^{2(1-f)}}; \quad \rho_{de} = \frac{3f^2}{n^2t^{2(1-f)}} - \frac{\rho_i}{a_0^3 \exp\left(\frac{3}{n}t^f\right)}; \\ w_{de} &= a_0^3 \left(\frac{2nf(1-f)t^{-f} - 3f^2}{3a_0^3f^2 - \rho_i n^2t^{2(1-f)} \exp\left(-\frac{3}{n}t^f\right)} \right); \\ \rho_{de} + 3p_{de} &= \frac{6f(1-f)}{nt^{2-f}} - \frac{6f^2}{n^2t^{2(1-f)}} - \frac{\rho_i}{a_0^3 \exp\left(\frac{3}{n}t^f\right)}; \quad \rho_{de} + p_{de} = \frac{2f(1-f)}{nt^{2-f}} - \frac{\rho_i}{a_0^3 \exp\left(\frac{3}{n}t^f\right)}; \\ \dot{\phi}^2 &= \frac{\lambda f^3}{n^3t^{3(1-f)}} + \frac{2\lambda f^2(1-f)}{n^2t^{3-2f}} + \frac{2f(1-f)}{nt^{2-f}} - \frac{\rho_i}{a_0^3 \exp\left(\frac{3}{n}t^f\right)}; \\ V &= \frac{3\lambda f^3}{2n^3t^{3(1-f)}} - \frac{\lambda f^2(1-f)}{n^2t^{3-2f}} + \frac{3f^2}{n^2t^{2(1-f)}} - \frac{f(1-f)}{nt^{2-f}} - \frac{\rho_i}{2a_0^3 \exp\left(\frac{3}{n}t^f\right)}. \end{aligned} \quad (10)$$

Above set of solutions (10) indicates that such a model of the Universe admits an early deceleration, but during evolution it starts accelerating since strong energy condition is violated, $\rho_{de} + 3p_{de} < 0$. Further, the dark energy equation of state also admits the possibility of crossing the $w_\Lambda = -1$ line, since, transient violation of the weak energy condition, $\rho_{de} + p_{de} < 0$ is seemingly possible. Finally, the equation of state asymptotically touches the $w_\Lambda = -1$ line from above and behaves as cosmological constant. To show such behavior graphically, let us express the state parameter w_{de} in terms of the red-shift parameter z which is defined as,

$$1 + z = \frac{a(t_o)}{a(t)} = \exp\left[\frac{1}{n}(t_o^f - t^f)\right],$$

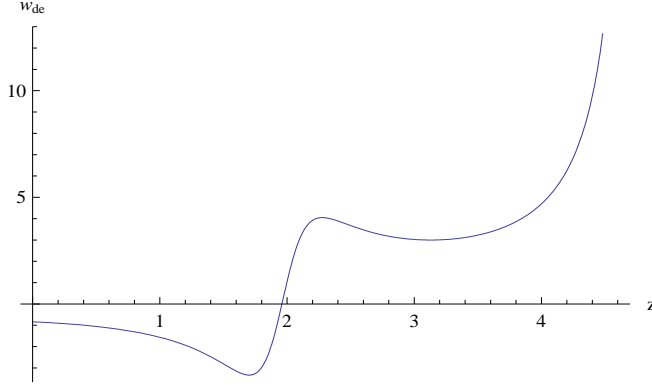


Figure 1: State parameters $w_{de}(z)$ has been plotted against the red-shift parameter z , (with, $f = 0.5, h = 0.66, t_0 = 13 \text{ Gyr}, \Omega_{mo} = 0.26$). Smooth double crossing of the Cosmological constant barrier is observed at sufficiently later epoch, $z \approx 1.92$ from above and $z \approx 0.39$ from below.

where, $a(t_o)$ is the present value of the scale factor, while $a(t)$ is that value at some arbitrary time t , when the light was emitted from a cosmological source. Thus,

$$t^f = t_o^f - n \ln(1 + z). \quad (11)$$

In view of equation (11) w_{de} can be expressed as,

$$w_{de} = \left(\frac{(2nf(1-f) - 3f^2[t_o^f - n \ln(1+z)])}{3f^2[t_o^f - n \ln(1+z)] - \rho_i n^2[t_o^f - n \ln(1+z)]^{\frac{2-f}{f}} \exp(-\frac{3}{n}[t_o^f - n \ln(1+z)])} \right), \quad (12)$$

where, a_o has been set equal to one without any loss of generality. Let us now choose $f = 0.5$. The motivation of choosing such a value of f is twofold. Primarily, it is impossible to find an explicit form of the potential $V = V(\phi)$, otherwise. Further, since n has the dimension of t^f , so the parameter n^2 gets a comfortable dimension of time. If we now take up some more numbers, like the present value of the Hubble parameter H_o^{-1} and the age of the Universe t_o as,

$$H_o^{-1} = \frac{9.78}{h} \text{ Gyr}, \quad t_o = 13 \text{ Gyr},$$

then, for $h = 0.66$, n can be found from the ansatz (9) as $n = 2.0552$. Further taking the present value of the matter density parameter $\Omega_{mo} = 0.26$, we find in view of solution (10),

$$\Omega_{mo} = \frac{\rho_{mo}}{\rho_{co}} = \rho_i \left(\frac{H_o^{-2} \exp(-\frac{3}{n}t_o^f)}{3} \right) = 0.26,$$

where, ρ_{mo} and ρ_{co} are the present values of the matter density and the critical density respectively. Thus, we find,

$$n^2 \rho_i = 2.897.$$

Noting that in this model the red-shift parameter does not go beyond the value $z = 4.78$, we plot the dark energy equation of state parameter (w_{de}) versus the red-shift parameter (z) in figure (1). It is apparent that the phantom divide line ω_Λ has been crossed twice, once from above at $z \approx 1.92$ and then from below at $z \approx 0.39$. Such double crossing of the phantom divide line is devoid of any sort of pathological behaviour.

To check how far our present model fits with the standard Λ CDM model, we also make the luminosity-redshift and distance modulus-redshift plots. For Λ CDM model the relation between the luminosity distance and redshift is the following,

$$H_0 dL = (1+z) \int_0^z \frac{dz}{0.74 + 0.26(1+z)^3},$$

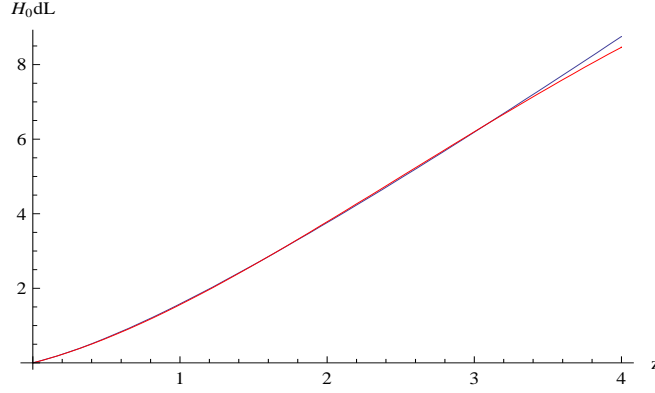


Figure 2: The fit is almost perfect up to $z = 3.5$. A little discrepancy is observed there after as Λ CDM model (blue) slightly takes over the present model (red).

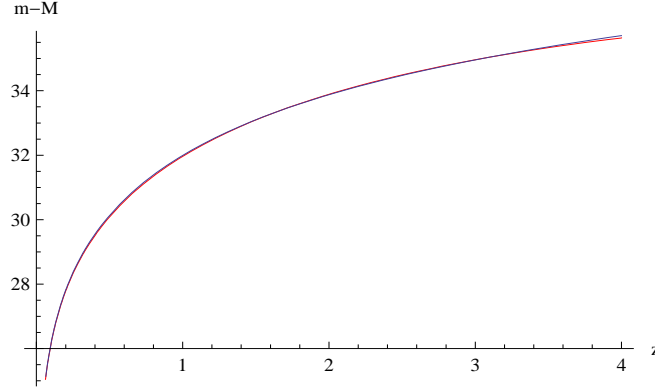


Figure 3: The fit is absolutely perfect and the two models (blue for Λ CDM and red for the present) are practically indistinguishable.

while the relation corresponding to the present model is,

$$H_0 dL = \frac{1+z}{\sqrt{t_0}} \int_0^z [\sqrt{t_0} - n \ln(1+z)] dz,$$

with $t_0 = 13$ Gyr., and $n = 2.055$. The plot (figure 2) shows a perfect fit between the two models up to $z = 3.5$. There is a little discrepancy there after. Since, luminosity distance has already been expressed as a function of redshift, so the relation between distance modulus and redshift may be found in view of the following equation,

$$m - M = 5 \log_{10} \left(\frac{dL}{Mpc} \right) + 25,$$

where, m and M are the apparent and absolute bolometric magnitudes respectively. However, since we use $H_0 dl$ instead, so our relation is slightly modified as,

$$m - M = 5 \log_{10}(DL) + 31,$$

where, $DL = H_0 dL$. The plot (figure 3) demonstrates that the two models are practically indistinguishable.

Now we can proceed to make some even more comfortable choice of the parameters of the theory, like $f = 0.5$, and $t_0 = 13$ Gyr. as before, but with $n = 2$, for which $H_0^{-1} = 14.42$ Gyr., in view of ansatz (9), which corresponds to $h \approx 0.68$. The scale factor now has a convenient form as $a = \exp \frac{\sqrt{t}}{2}$. With these values one can find,

$$n^2 \rho_i = 3.09,$$

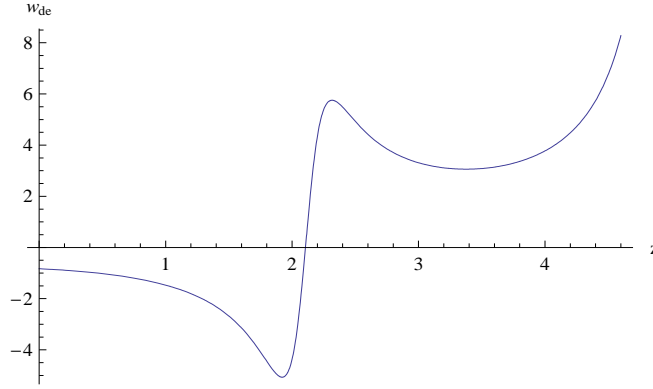


Figure 4: State parameters $w_{de}(z)$ has been plotted against the red-shift parameter z , (with, $f = 0.5, h \approx 0.68, t_0 = 13 \text{ Gyr.}, \Omega_{mo} = 0.24$). Smooth double crossing of the Cosmological constant barrier is observed at sufficiently later epoch, $z \approx 2.085$ from above and $z \approx 0.46$ from below.

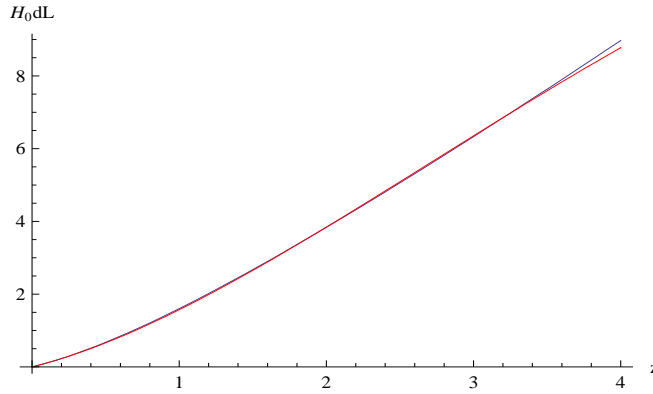


Figure 5: The fit is perfect up-to $z = 3.6$. The Λ CDM model (blue) slightly overtakes the present model (red) there after.

for $\Omega_{mo} = 0.24$. The plot (fig.4) is almost the same as before with transient double crossing. Other plots viz., luminosity distance versus redshift (in figure 5) and distance modulus versus redshift (in figure 6) show even better fit than the earlier one, with the standard Λ CDM model.

Thus, we observe that with the age $t_0 = 13 \text{ Gyr.}$, and $0.66 \leq h \leq 0.68$, such transient crossing of the phantom divide line is permissible for the present value of dark energy density parameter $\Omega_{de}|_{\text{present}} \geq 0.74$. In order to consider some higher value of the age of the Universe, $t_0 = 13.73 \text{ Gyr.}$ (say) as suggested by Spergel et al[24], either one has to go to almost the lowest limiting value of $h \approx 0.61$ [25] or one has to accept much higher value of the present dark energy density parameter $\Omega_{de}|_{\text{present}} > 0.78$, otherwise, the state parameter versus redshift plot shows certain discontinuities. We certainly remember that in order to simplify the field equations considerably, we have made one important assumption, viz., in equation (5). Relaxing this assumption one might get rid of such discontinuities, as well. This we pose to study in a future communication.

So far, we remain silent about the form of the potential. It is simply because, despite the most convenient choice of the parameter, $f = 0.5$, it is still impossible to find an analytical solution for ϕ , in view of the solution (10). As a result, the form of the potential as a function of ϕ remains obscure. However, we can plot the potential as a function of time, by choosing our second case, $n = 2$, for further simplification. It is important to note that though the results sofar obtained, are independent of the value and signature of λ , the form of the potential depends largely on it. In the following we make three such plots (taking the help of "Manipulation" programme of Mathematica 6) to show how the form of the potential changes with different values (starting from negative to large positive) of λ .

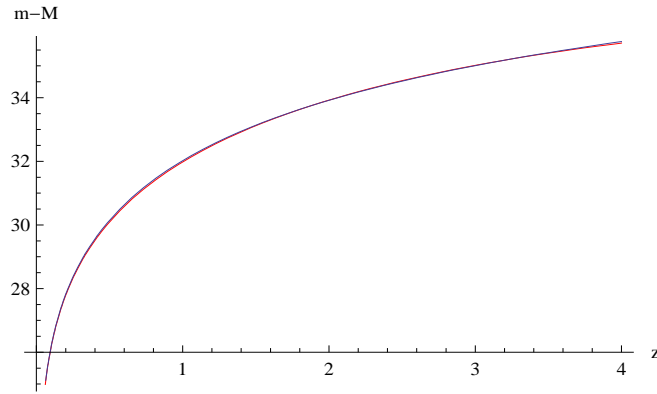


Figure 6: The fit is absolutely perfect and there is practically no way to distinguish the Λ CDM model (blue) with the present one (red).

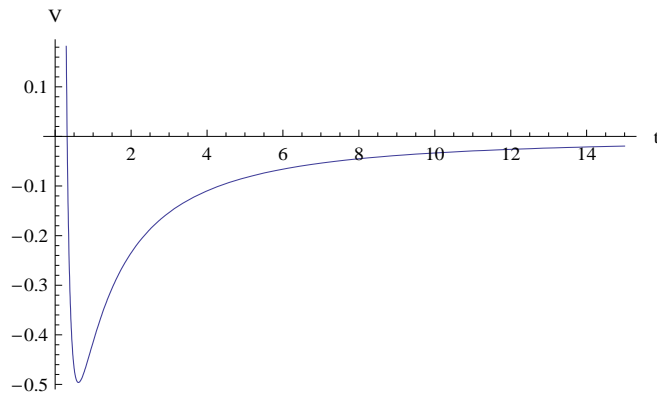


Figure 7: The form of the potential as a function of time for $\lambda \leq -9$. Note that it remains negative throughout the evolution.

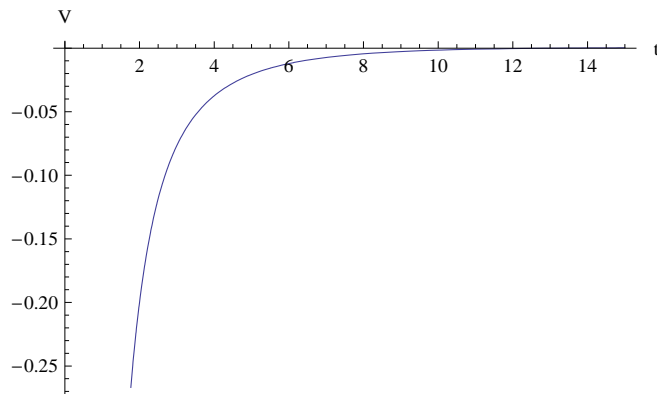


Figure 8: The form of the potential for $\lambda \approx 65$. Note that it is zero at the present epoch but tends to grow in the future.

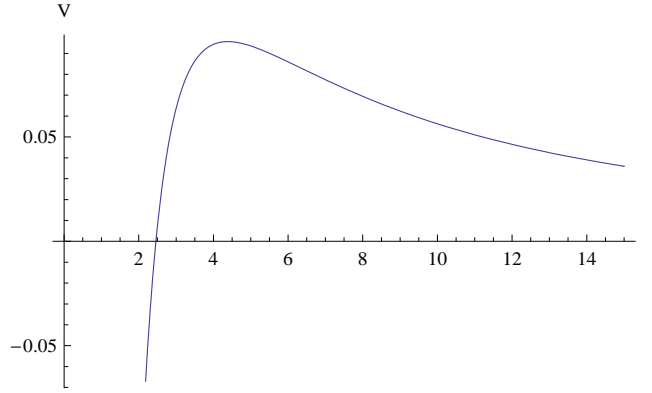


Figure 9: The form of the potential for $\lambda = 200$. Note that the form is appreciably different.

3 Concluding remarks

Altogether we have obtained a late time transient crossing of the phantom divide line first from above and more recently from below, starting from the inclusion of a Gauss-Bonnet-dilatonic scalar coupling term in the standard Einstein-Hilbert action in four dimension. Since the crossing is transient, so such double crossing is free from any sort of pathological behaviour both at the classical [6] and at the quantum mechanical [7] levels. The striking feature of the model lies in its indistinguishability with the standard Λ CDM model, in terms of the luminosity-redshift and more precisely for distance modulus-redshift curves. To identify between the two models we therefore require to observe dark energy equation of state w_{de} independently. If w_{de} is truly found to be dynamical and has really encountered a recent crossing of the phantom divide line, then only we can definitely distinguish the standard Λ CDM model with the present one. It is highly interesting to learn that smooth transient crossing of the phantom divide line is allowed for both negative and positive ($\lambda \leq 0$) type of Gauss-Bonnet-scalar interaction. Figures (7), (8) and (9) also reveal that it is true even for different forms of the potential. Such transient crossing independent of the value of λ also signals that it might be possible to carry out the same treatment even for a single scalar field model.

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